Student's *t* Distribution

Lecture 34
Section 10.2

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Outline

1. TI-83 Data Option
2. Student's *t* Distribution
3. Degrees of Freedom
4. *t* Probabilities
5. Assignment

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Example

The `Data` Option

Example (TI-83 `Data` Option)

- A large number of students at a high school take the AP test in Statistics.
- A random sample of 36 students reveals the following scores:

Score	Frequency
1	2
2	5
3	16
4	11
5	2

Example

The `Data` Option

Example (TI-83 `Data` Option)

- Assume that the population standard deviation is $\sigma = 1$.
- Use the TI-83 to test the hypothesis that the average score for the high school is above 3.

Hypothesis Testing on the TI-83

The Data Option

TI-83 Hypothesis Testing for the Mean (Data Option)

- Enter the data (all 36 scores) into list L_1.
- Enter the hypothetical mean μ_0.
- Enter σ.
- Identify the list that contains the data (L_1).
- Skip Freq.
- Select the alternative hypothesis.
- Select Calculate and press ENTER.

Hypothesis Testing on the TI-83

The Data Option

TI-83 Hypothesis Testing for the Mean (Data Option)

OR

- Enter the 5 distinct scores into list L_1.
- Enter the 5 frequencies into list L_2.
- Enter the hypothetical mean μ_0.
- Enter σ.
- Identify the list that contains the data (L_1).
- For Freq, enter L_2.
- Select the alternative hypothesis.
- Select Calculate and press ENTER.

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- Last time, we made a very important, but unrealistic, assumption: The value of σ was known.
- What happens if σ is not known?
- We have to use s as an approximate the value of σ.
- Does that have any ramifications?
- Yes!

What if σ is Unknown?

- Instead of the test statistic

$$\frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}},$$

we will use the statistic

$$\frac{\overline{x} - \mu_0}{s/\sqrt{n}}.$$

- Does this statistic have a normal distribution?

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  we will use the statistic

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- Does this statistic have a normal distribution?
- No.

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  we will use the statistic

$$\frac{\overline{x} - \mu_0}{s/\sqrt{n}}.$$

- Does this statistic have a normal distribution?
- No.
- So rather than call it z, it is called t.

Student's t Distribution

Definition (Student's t distribution)

If the population from which we sample is normal, then the statistic

$$\frac{\overline{x} - \mu}{s/\sqrt{n}}$$

has a distribution called Student's t distribution. Therefore, we will write

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}}.$$

- If the population from which we sample is not normal, then we should not use the t distribution.

Student's *t* Distribution

- The *t* distribution was discovered by W. S. Gosset in 1908.
- See the MathWorld article Student's *t*-Distribution

The *t* Distribution

- The shape of the *t* distribution is very similar to the shape of the standard normal distribution.
- It is
  - Symmetric
  - Unimodal
  - Centered at 0.
- But it is wider than the standard normal.
- That is because of the additional variability introduced by using *s* instead of σ.

The *t* Distribution

- Furthermore, the shape of the *t* distribution depends on the sample size.
- It has a slightly different shape for each sample size.
- When *n* is small, *s* has a high variability, so the shape is wider.
- When *n* is large, *s* is not so variable, so the shape is not as wide.

The t Distribution

- However, as n gets larger and larger, the shape approaches the standard normal z.
- In fact, if $n \geq 30$, then the t distribution is almost exactly standard normal.

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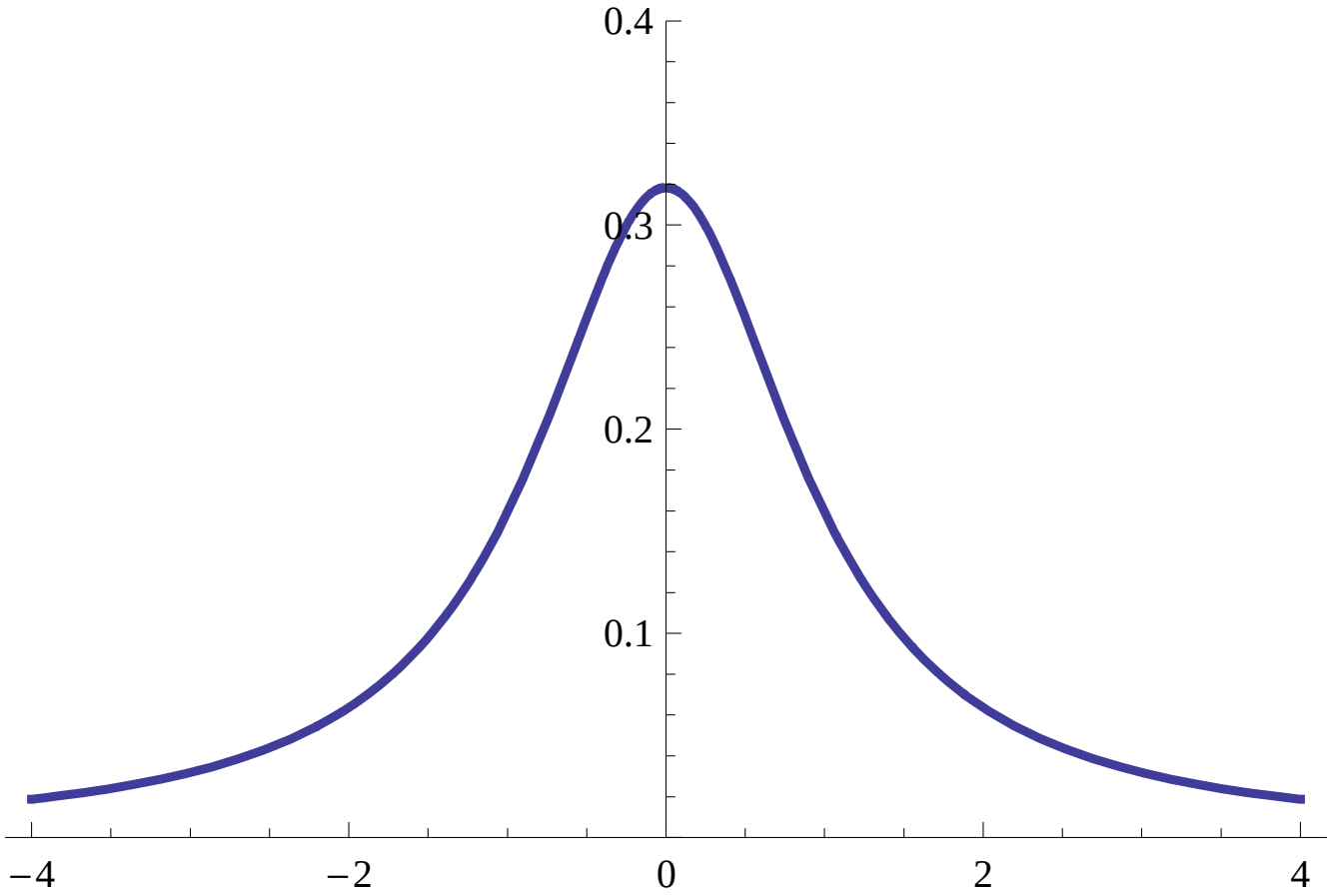
Degrees of Freedom

Definition (Degrees of freedom)

If the sample size is n, then t is said to have $n - 1$ degrees of freedom.

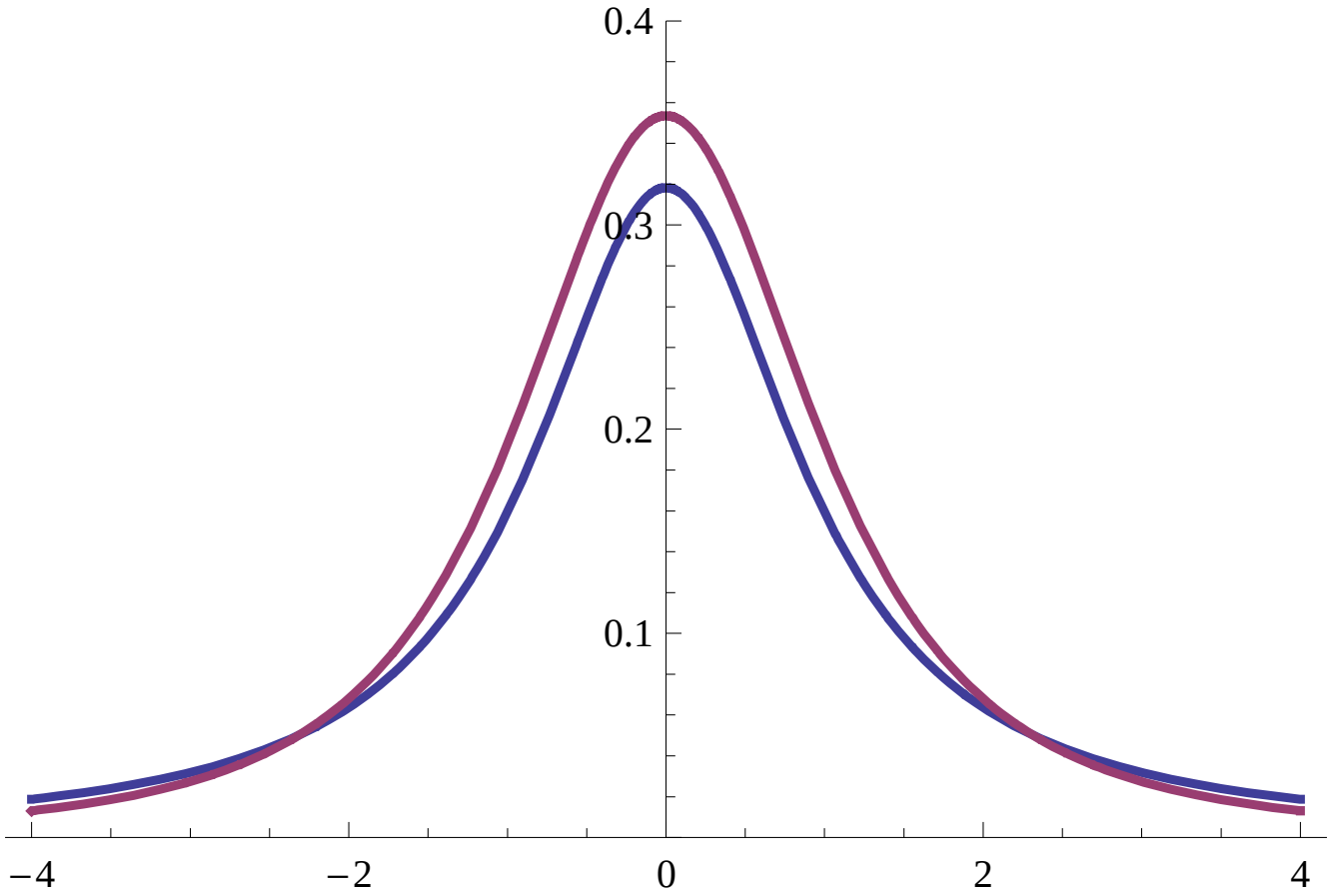
- We use *df* to denote “degrees of freedom."
- We will use the notation t_5 to denote the t distribution with 5 degrees of freedom (i.e., sample size 6).
- The book uses the notation $t(5)$.

Standard Normal vs. t Distribution



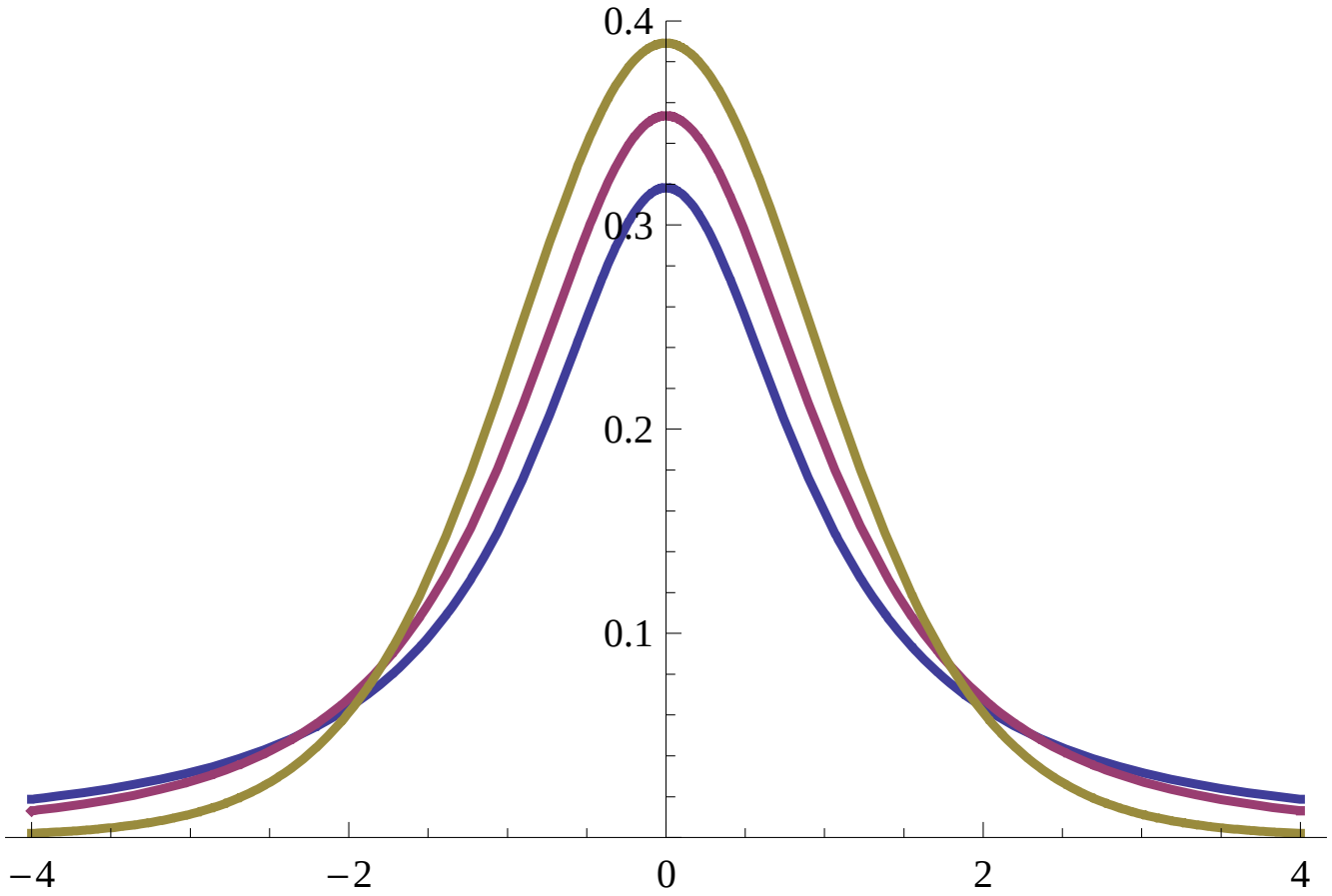
The distribution of t_1.

Standard Normal vs. t Distribution



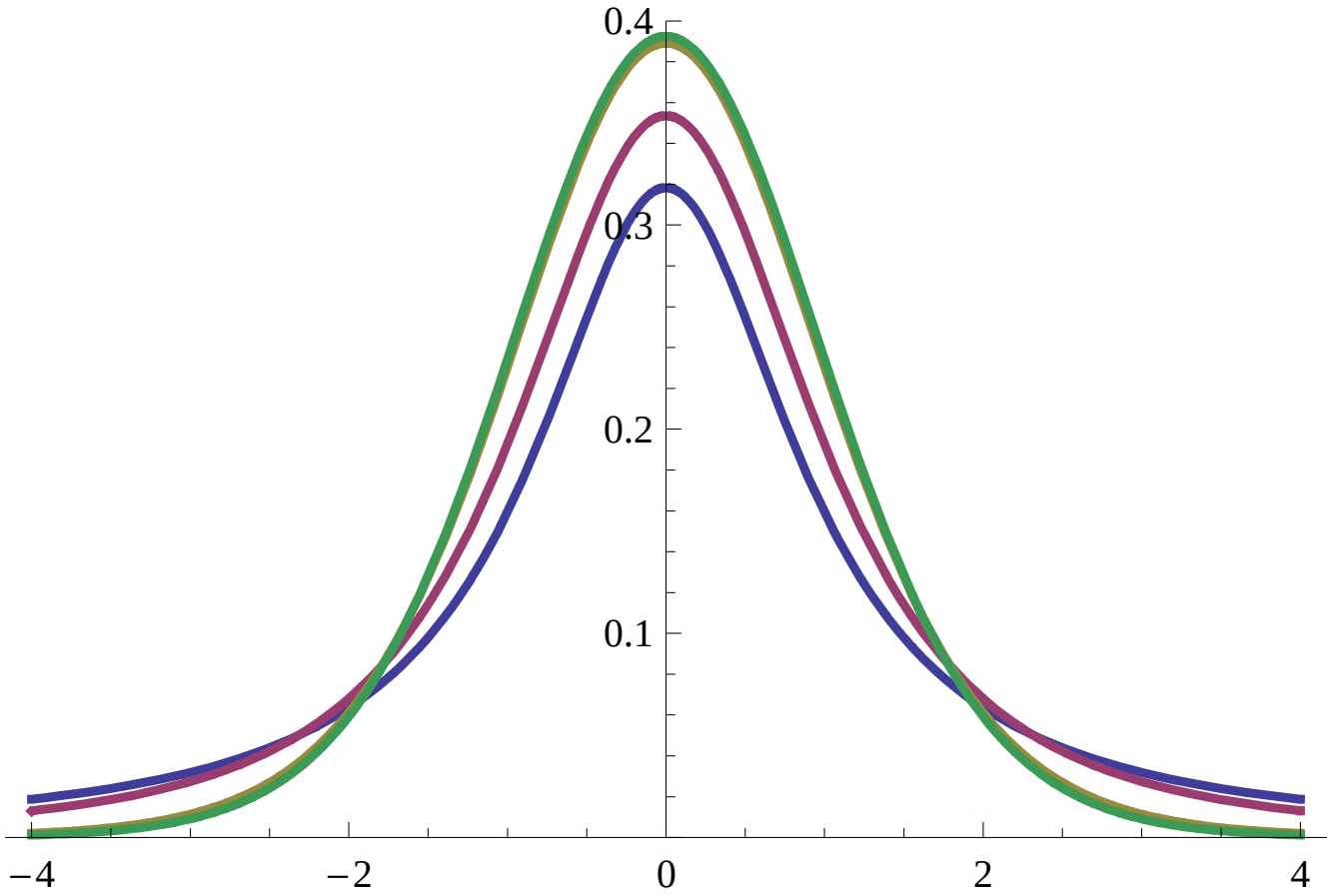
The distribution of t_2.

Standard Normal vs. t Distribution



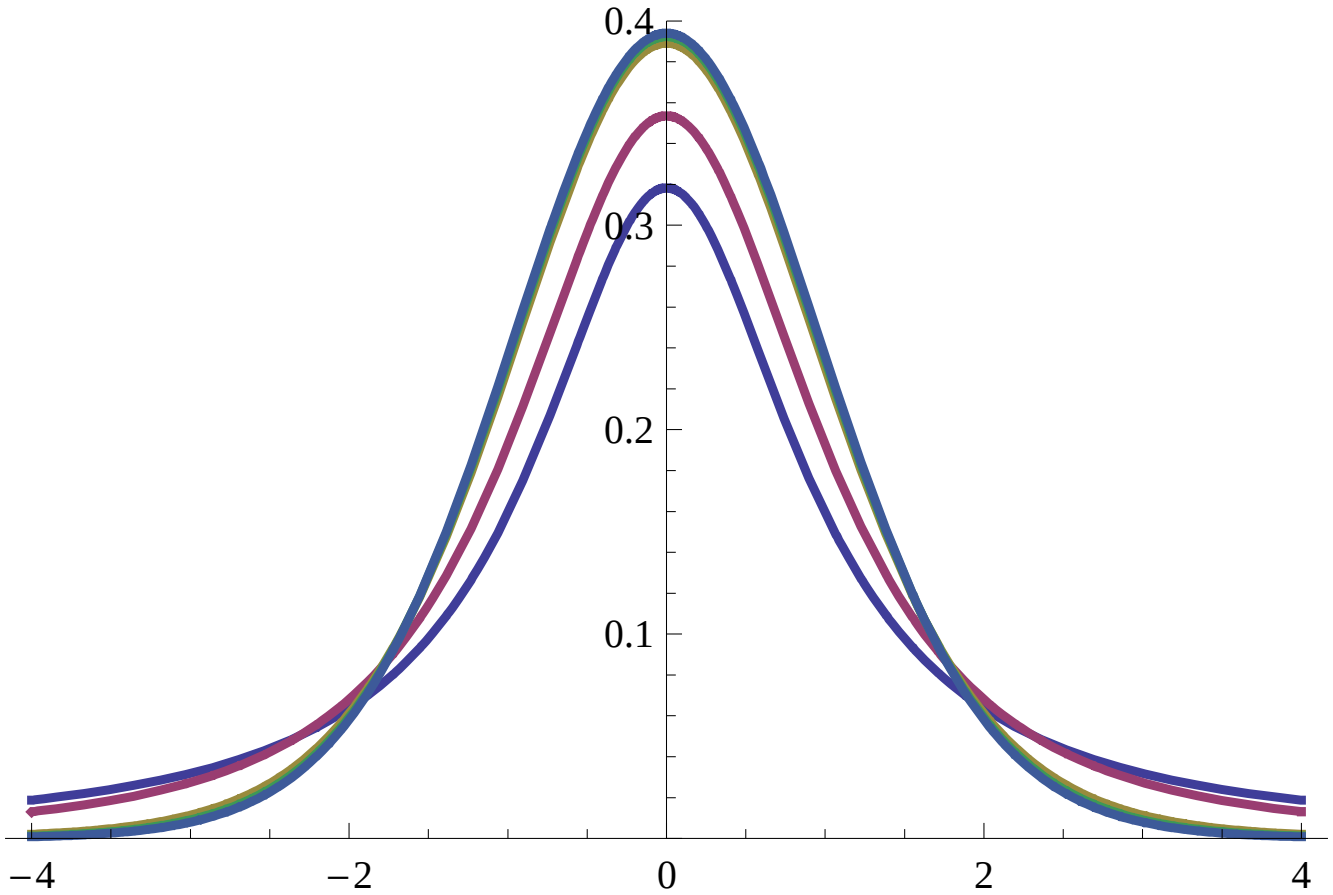
The distribution of t_{10}.

Standard Normal vs. t Distribution



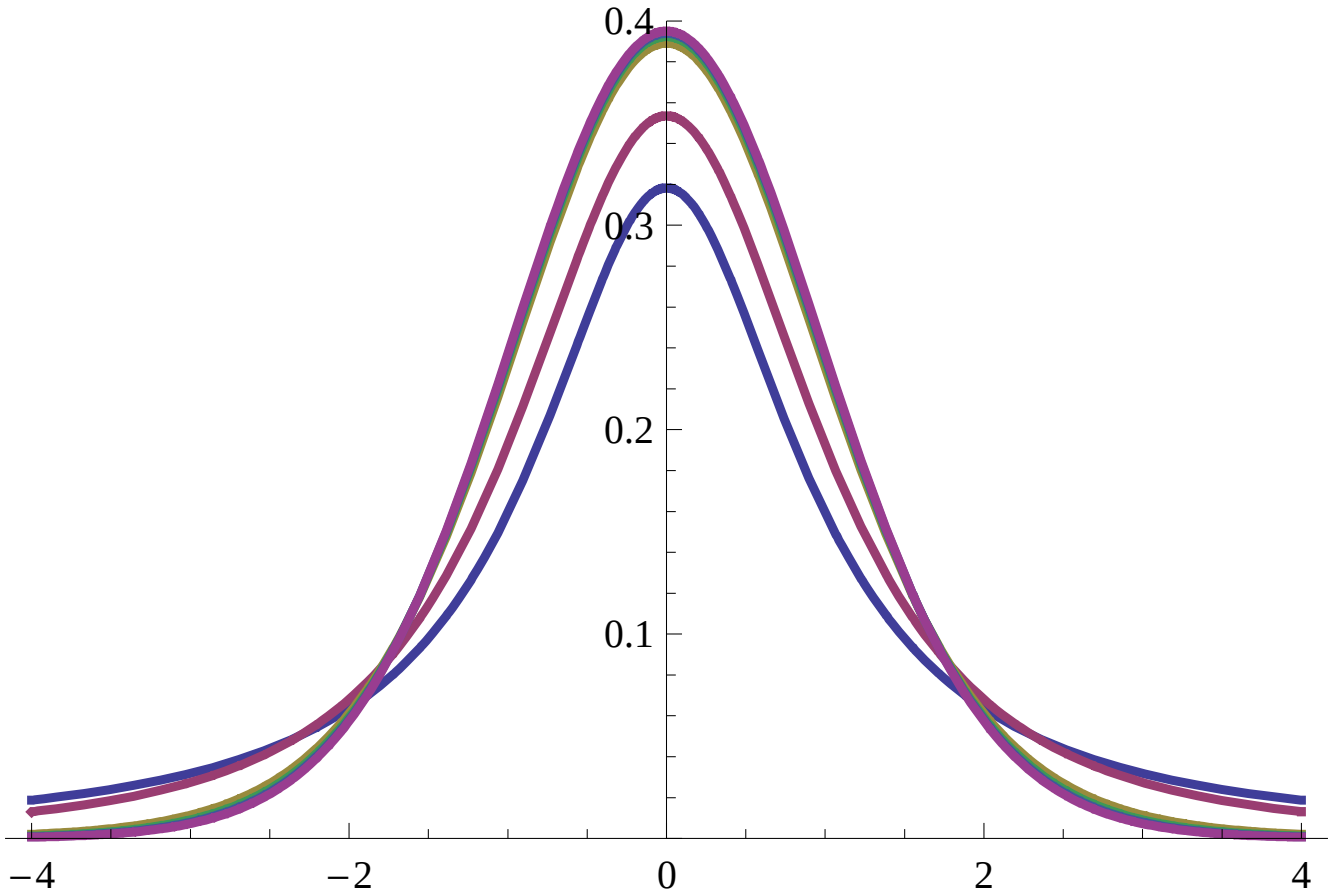
The distribution of t_{15}.

Standard Normal vs. t Distribution



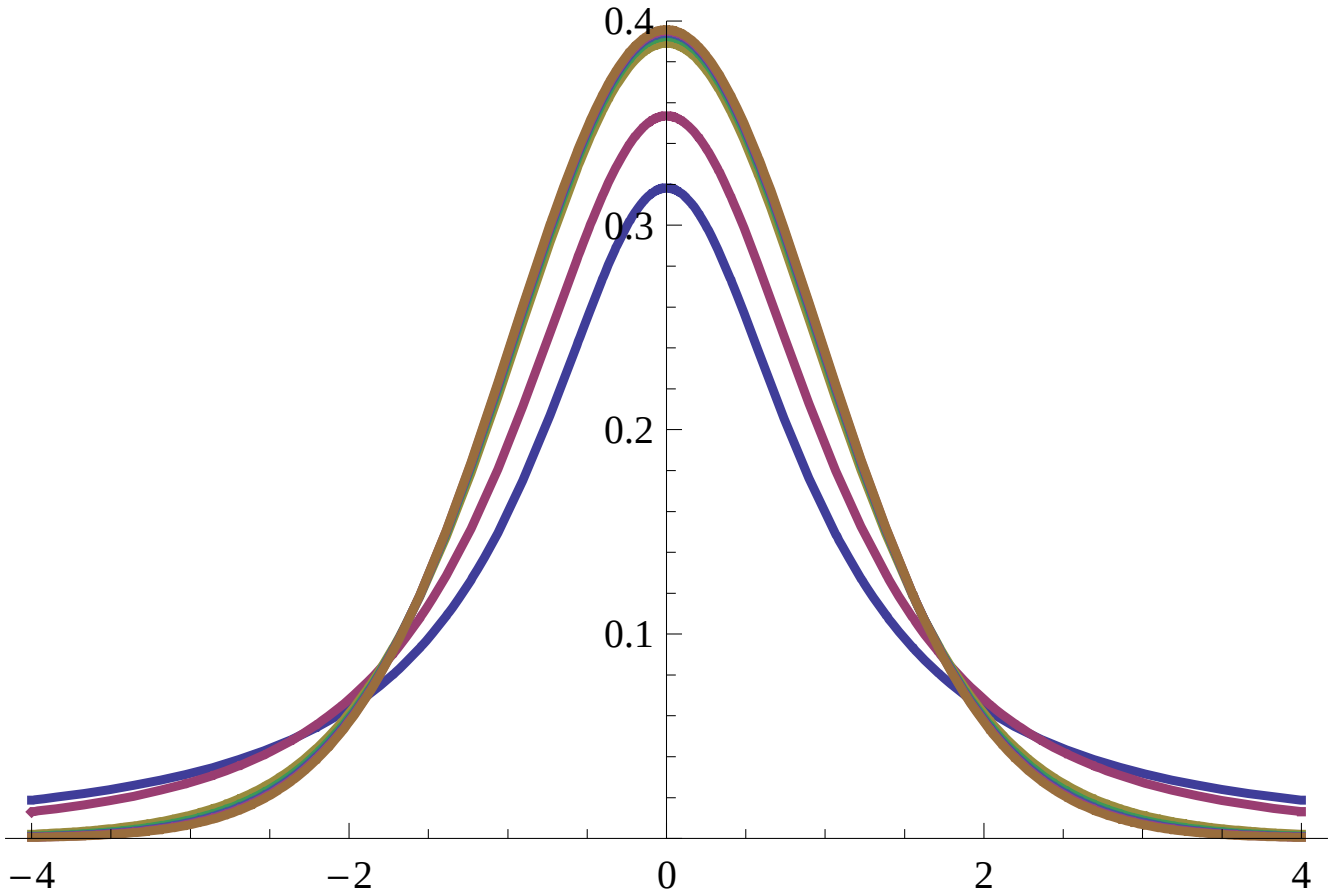
The distribution of t_{20}.

Standard Normal vs. t Distribution



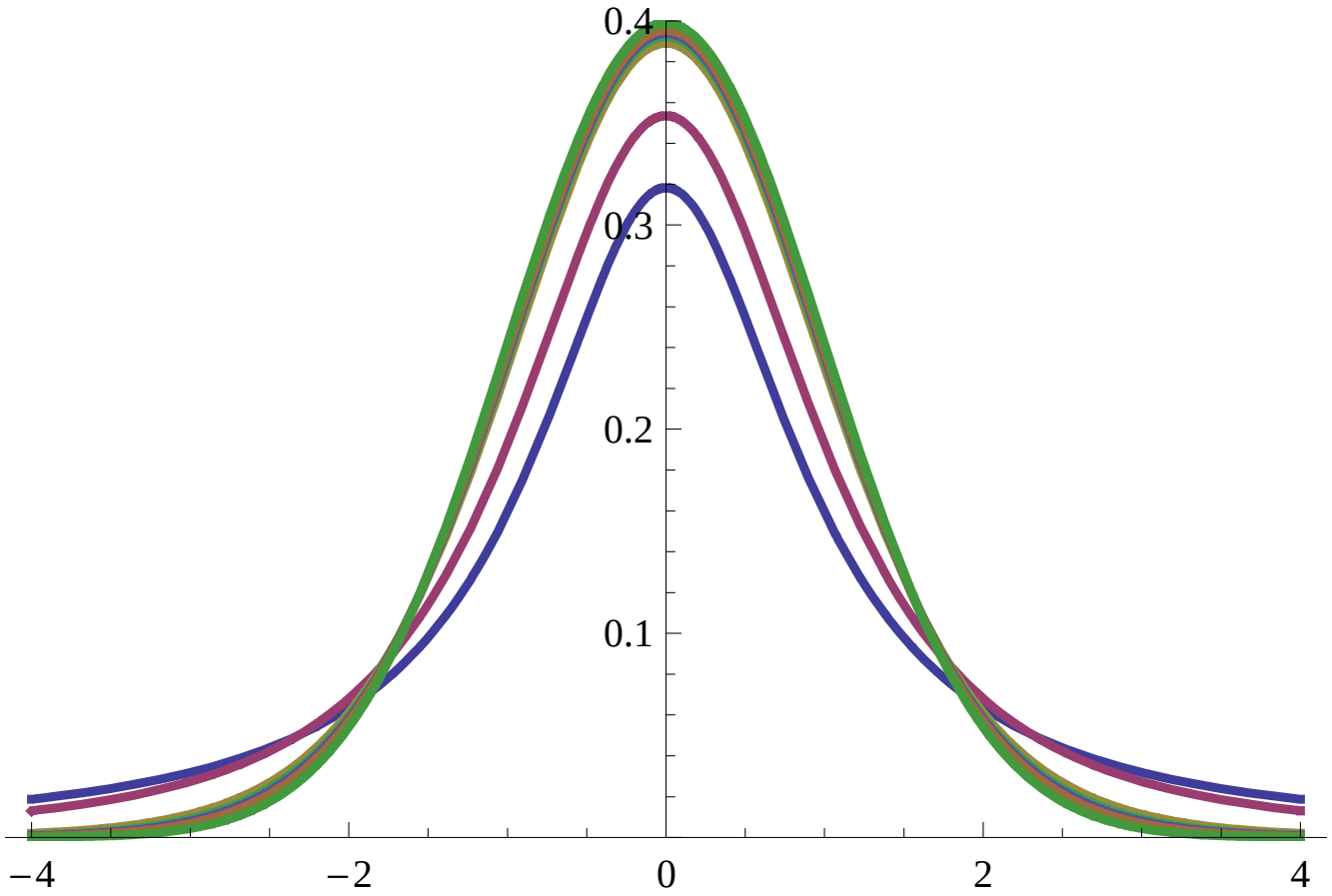
The distribution of t_{25}.

Standard Normal vs. t Distribution



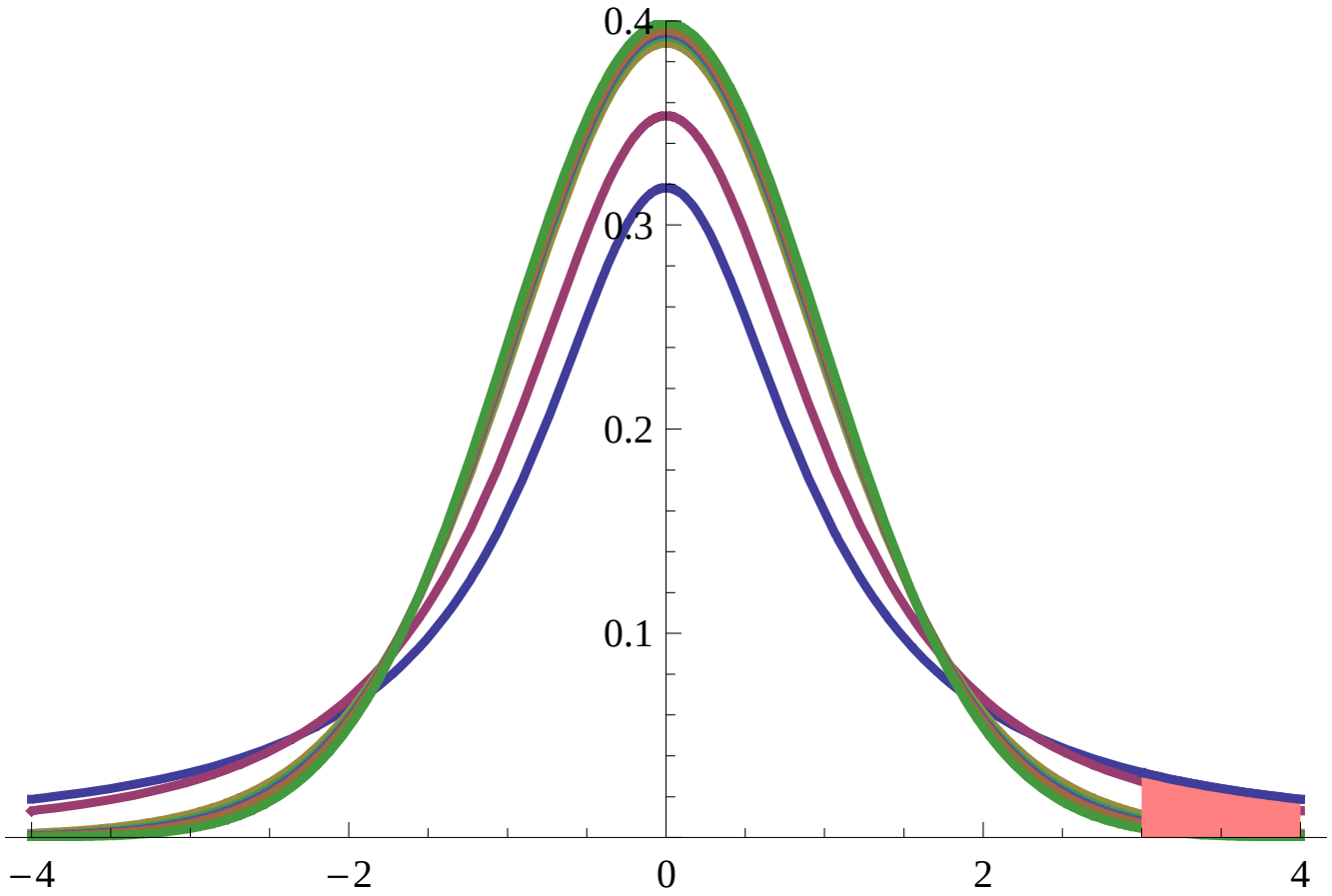
The distribution of t_{30}.

Standard Normal vs. t Distribution



The distribution of z.

Standard Normal vs. *t* Distribution



Area of rightmost tail.

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t Probabilities

- To find probabilities concerning t, use the TI-83 function `tcdf`.
- `tcdf` takes three parameters:
  - The left endpoint (sometimes `-E99`).
  - The right endpoint (sometimes `E99`).
  - The number of degrees of freedom.
- For example, $P(-1 < t_{10} < 1)$ is given by

```
tcdf(-1,1,10).
```

t Probabilities

Example (t Probabilities)

- Find $P(t > 3)$ with $df = 1$.
- Find $P(t > 3)$ with $df = 2$.
- Find $P(t > 3)$ with $df = 10$.
- Find $P(t > 3)$ with $df = 30$.
- Find $P(z > 3)$.

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Assignment

Homework

- Read Section 10.2, pages 621 - 633.
- Let's Do It! 10.3, 10.4, 10.5.
- Exercises 7, 8, page 633.